AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

1. (Currently amended) A method for using a computer system to solve a global inequality constrained optimization problem specified by a function f and a set of inequality constraints $p_i(\mathbf{x}) \leq 0$ $(i=1,\dots,m)$, wherein f and p_i are scalar functions of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots x_n)$, the method comprising:

receiving a representation of the function f and the set of inequality constraints at the computer system;

storing the representation in a memory within the computer system;

performing an interval inequality constrained global optimization process to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$ subject to the set of inequality constraints;

wherein performing the interval global optimization process involves,

applying term consistency to the set of inequality constraints over a sub-box $\mathbf{X}$, and

excluding any portion of the sub-box $\mathbf{X}$ that is proved to be in violation of at least one member of the set of inequality constraints; and

recording the guaranteed bounds in the computer system memory;

wherein applying term consistency involves:

symbolically manipulating an equation within the computer system to solve for a term, $g(x'_j)$, thereby producing a modified

equation $g(x'_j) = h(\mathbf{x})$, wherein the term $g(x'_j)$ can be analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;

substituting the sub-box **X** into the modified equation to produce the equation $g(X'_j) = h(\mathbf{X})$;

solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and

intersecting X'_j with the j-th element of the sub-box **X** to produce a new sub-box $\mathbf{X}^+$;

wherein the new sub-box $\mathbf{X}^+$ contains all solutions of the equation within the sub-box **X**, and wherein the size of the new sub-box $\mathbf{X}^+$ is less than or equal to the size of the sub-box **X**.

2. (Previously presented) The method of claim 1, further comprising:

linearizing the set of inequality constraints to produce a set of linear inequality constraints with interval coefficients that enclose the nonlinear constraints;

preconditioning the set of linear inequality constraints through additive linear combinations to produce a preconditioned set of linear inequality constraints;

applying term consistency to the set of preconditioned linear inequality constraints over the sub-box **X**, and

excluding any portion of the sub-box **X** that violates any member of the set of preconditioned linear inequality constraints.

3. (Original) The method of claim 2, further comprising:

keeping track of a least upper bound *f_bar* of the function $f(\mathbf{x})$ at a feasible point **x** wherein $p_i(\mathbf{x}) \leq 0$ $(i=1,\ldots,m)$; and

including $f(\mathbf{x}) \leq f\_bar$ in the set of inequality constraints prior to linearizing the set of inequality constraints.

4. (Original) The method of claim 2, further comprising removing from consideration any inequality constraints that are not violated by more than a specified amount for purposes of applying term consistency prior to linearizing the set of inequality constraints.

5. (Previously presented) The method of claim 1, wherein performing the interval global optimization process involves:

keeping track of a least upper bound *f_bar* of the function $f(\mathbf{x})$ at a feasible point $\mathbf{x}$;

removing from consideration any sub-box for which $f(\mathbf{x}) > f\_bar$;

applying term consistency to the *f_bar* inequality $f(\mathbf{x}) \leq f\_bar$ over the sub-box $\mathbf{X}$; and

excluding any portion of the sub-box $\mathbf{X}$ that violates the *f_bar* inequality.

6. (Previously presented) The method of claim 1, wherein if the sub-box $\mathbf{X}$ is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1,\ldots,n$), performing the interval global optimization process involves:

determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ $(i=1,\ldots,n)$;

removing from consideration any sub-box for which $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that the sub-box does not include an extremum of $f(\mathbf{x})$; and

applying term consistency to each component $g_i(\mathbf{x})=0$ $(i=1,\ldots,n)$ of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the sub-box $\mathbf{X}$; and

excluding any portion of the sub-box $\mathbf{X}$ that violates any component of $\mathbf{g}(\mathbf{x})=\mathbf{0}$.

7. (Previously presented) The method of claim 1, wherein if the sub-box **X** is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1,\ldots,n$), performing the interval global optimization process involves:

determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1,\ldots,n$) of the Hessian of the function $f(\mathbf{x})$;

removing from consideration any sub-box for which $H_{ii}(\mathbf{x})$ a diagonal element of the Hessian over the sub-box **X** is always negative, indicating that the function f is not convex over the sub-box **X** and consequently does not contain a global minimum within the sub-box **X**;

applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1,\ldots,n$) over the sub-box **X**; and

excluding any portion of the sub-box **X** that violates a Hessian inequality.

8. (Previously presented) The method of claim 1, wherein if the sub-box **X** is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1,\ldots,n$), performing the interval global optimization process involves:

performing the Newton method, wherein performing the Newton method involves,

computing the Jacobian $\mathbf{J}(\mathbf{x},\mathbf{X})$ of the gradient of the function f evaluated with respect to a point **x** over the sub-box **X**,

computing an approximate inverse **B** of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$,

using the approximate inverse **B** to analytically determine the system $\mathbf{Bg}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1,\ldots,n$);

applying term consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ ($i=1,\ldots,n$) for each variable x_i ($i=1,\ldots,n$) over the sub-box **X**; and

excluding any portion of the sub-box **X** that violates a component.

9 (Canceled).

10. (Original) The method of claim 1, further comprising performing the Newton method on the John conditions.

11. (Currently amended) A computer-readable storage medium storing instructions that when executed by a computer cause the computer to perform a method for using a computer system to solve a global inequality constrained optimization problem specified by a function *f* and a set of inequality constraints $p_i(\mathbf{x}) \leq 0\ (i=1,\ldots,m)$, wherein *f* is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \ldots x_n)$, the method comprising:

receiving a representation of the function *f* and the set of inequality constraints at the computer system;

storing the representation in a memory within the computer system;

performing an interval inequality constrained global optimization process to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$ subject to the set of inequality constraints;

wherein performing the interval global optimization process involves,

applying term consistency to the set of inequality constraints over a sub-box **X**, and

excluding any portion of the sub-box **X** that is proved to be in violation of at least one member of the set of inequality constraints; and

recording the guaranteed bounds in the computer system memory;

wherein applying term consistency involves:

symbolically manipulating an equation within the computer system to solve for a term, $g(x'_j)$, thereby producing a modified equation $g(x'_j) = h(\mathbf{x})$, wherein the term $g(x'_j)$ can be analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;

substituting the sub-box **X** into the modified equation to produce the equation $g(X'_j) = h(\mathbf{X})$;

solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and

intersecting X'_j with the j-th element of the sub-box **X** to produce a new sub-box $\mathbf{X}^+$;

wherein the new sub-box $\mathbf{X}^+$ contains all solutions of the equation within the sub-box **X**, and wherein the size of the new sub-box $\mathbf{X}^+$ is less than or equal to the size of the sub-box **X**.

12. (Previously presented) The computer-readable storage medium of claim 11, wherein the method further comprises:

linearizing the set of inequality constraints to produce a set of linear inequality constraints with interval coefficients that enclose the nonlinear constraints;

preconditioning the set of linear inequality constraints through additive linear combinations to produce a preconditioned set of linear inequality constraints;

applying term consistency to the set of preconditioned linear inequality constraints over the sub-box **X**, and

excluding any portion of the sub-box **X** that violates any member of the set of preconditioned linear inequality constraints.

13. (Original) The computer-readable storage medium of claim 12, wherein the method further comprises:

keeping track of a least upper bound *f_bar* of the function $f(\mathbf{x})$ at a feasible point $\mathbf{x}$ wherein $p_i(\mathbf{x}) \leq 0$ $(i=1,\ldots,m)$; and

including $f(\mathbf{x}) \leq$ *f_bar* in the set of inequality constraints prior to linearizing the set of inequality constraints.

14. (Original) The computer-readable storage medium of claim 12, wherein the method further comprises removing from consideration any inequality constraints that are not violated by more than a specified amount for purposes of applying term consistency prior to linearizing the set of inequality constraints.

15. (Previously presented) The computer-readable storage medium of claim 11, wherein performing the interval global optimization process involves:

keeping track of a least upper bound *f_bar* of the function $f(\mathbf{x})$ at a feasible point $\mathbf{x}$;

removing from consideration any sub-box for which $f(\mathbf{x}) >$ *f_bar*;

applying term consistency to the *f_bar* inequality $f(\mathbf{x}) \leq$ *f_bar* over the sub-box $\mathbf{X}$; and

excluding any portion of the sub-box $\mathbf{X}$ that violates the *f_bar* inequality.

16. (Previously presented) The computer-readable storage medium of claim 11, wherein if the sub-box $\mathbf{X}$ is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1,\ldots,n$), performing the interval global optimization process involves:

determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ $(i=1,\ldots,n)$;

removing from consideration any sub-box for which $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that the sub-box does not include an extremum of $f(\mathbf{x})$; and

applying term consistency to each component $g_i(\mathbf{x})=0$ $(i=1,\ldots,n)$ of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the sub-box $\mathbf{X}$; and

excluding any portion of the sub-box $\mathbf{X}$ that violates any component of $\mathbf{g}(\mathbf{x})=\mathbf{0}$.

17. (Previously presented) The computer-readable storage medium of claim 11, wherein if the sub-box $\mathbf{X}$ is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1,\ldots,n$), performing the interval global optimization process involves:

determining diagonal elements $H_{ii}(\mathbf{x})$ $(i=1,\ldots,n)$ of the Hessian of the function $f(\mathbf{x})$;

removing from consideration any sub-box for which $H_{ii}(\mathbf{x})$ a diagonal element of the Hessian over the sub-box $\mathbf{X}$ is always negative, indicating that the function f is not convex over the sub-box $\mathbf{X}$ and consequently does not contain a global minimum within the sub-box $\mathbf{X}$;

applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ $(i=1,\ldots,n)$ over the sub-box $\mathbf{X}$; and

excluding any portion of the sub-box $\mathbf{X}$ that violates a Hessian inequality.

18. (Previously presented) The computer-readable storage medium of claim 11, wherein if the sub-box $\mathbf{X}$ is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1,\ldots,n$), performing the interval global optimization process involves:

performing the Newton method, wherein performing the Newton method involves,

computing the Jacobian $\mathbf{J}(\mathbf{x},\mathbf{X})$ of the gradient of the function f evaluated with respect to a point $\mathbf{x}$ over the sub-box $\mathbf{X}$,

computing an approximate inverse $\mathbf{B}$ of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$,

using the approximate inverse **B** to analytically determine the system $\mathbf{Bg(x)}$, wherein $\mathbf{g(x)}$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g(x)}$ includes components $g_i(\mathbf{x})$ $(i=1,\ldots,n)$;

applying term consistency to each component $(\mathbf{Bg(x)})_i = 0$ $(i=1,\ldots,n)$ for each variable x_i $(i=1,\ldots,n)$ over the sub-box **X**; and

excluding any portion of the sub-box **X** that violates a component.

19 (Canceled).

20. (Original) The computer-readable storage medium of claim 11, wherein the method further comprises performing the Newton method on the John conditions.

21. (Currently amended) An apparatus for using a computer system to solve a global inequality constrained optimization problem specified by a function f and a set of inequality constraints $p_i(\mathbf{x}) \leq 0$ $(i=1,\ldots,m)$, wherein f is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \ldots x_n)$, the apparatus comprising:

a receiving mechanism that is configured to receive a representation of the function f and the set of inequality constraints at the computer system;

a memory within the computer system for storing the representation;

a global optimizer that is configured to perform an interval inequality constrained global optimization process to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$ subject to the set of inequality constraints;

a term consistency mechanism within the global optimizer that is configured to,

apply term consistency to the set of inequality constraints over a sub-box **X**, and to

exclude any portion of the sub-box **X** that is proved to be in violation of at least one member of the set of inequality constraints; and

a recording mechanism that is configured record the guaranteed bounds in the computer system memory;

wherein the term consistency mechanism is configured to:

symbolically manipulate an equation within the computer system to solve for a term, $g(x'_j)$, thereby producing a modified equation $g(x'_j) = h(\mathbf{x})$, wherein the term $g(x'_j)$ can be analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;

substitute the sub-box **X** into the modified equation to produce the equation $g(X'_j) = h(\mathbf{X})$;

solve for $X'_j = g^{-1}(h(\mathbf{X}))$; and

intersect X'_j with the j-th element of the sub-box **X** to produce a new sub-box $\mathbf{X}^+$;

wherein the new sub-box $\mathbf{X}^+$ contains all solutions of the equation within the sub-box **X**, and wherein the size of the new sub-box $\mathbf{X}^+$ is less than or equal to the size of the sub-box **X**.

22. (Previously presented) The apparatus of claim 21, further comprising:

a linearizing mechanism that is configured to linearize the set of inequality constraints to produce a set of linear inequality constraints with interval coefficients that enclose the nonlinear constraints; and

a preconditioning mechanism that is configured to precondition the set of linear inequality constraints through additive linear combinations to produce a preconditioned set of linear inequality constraints;

wherein the term consistency mechanism is configured to,

apply term consistency to the set of preconditioned linear inequality constraints over the sub-box **X**, and to

exclude any portion of the sub-box **X** that violates any member of the set of preconditioned linear inequality constraints.

23. (Original) The apparatus of claim 22, wherein the global optimizer is configured to:

keep track of a least upper bound *f_bar* of the function $f(\mathbf{x})$ at a feasible point **x** wherein $p_i(\mathbf{x}) \leq 0\ (i=1,\ldots,m)$; and to

include $f(\mathbf{x}) \leq f\_bar$ in the set of inequality constraints prior to linearizing the set of inequality constraints.

24. (Original) The apparatus of claim 22, wherein the term consistency mechanism is configured to remove from consideration any inequality constraints that are not violated by more than a specified amount for purposes of applying term consistency prior to linearizing the set of inequality constraints.

25. (Previously presented) The apparatus of claim 21,

wherein the global optimizer is configured to,

keep track of a least upper bound *f_bar* of the function $f(\mathbf{x})$ at a feasible point **x**, and to

remove from consideration any sub-box for which $f(\mathbf{x}) > f\_bar$;

wherein the term consistency mechanism is configured to,

apply term consistency to the *f_bar* inequality $f(\mathbf{x}) \leq f\_bar$ over the sub-box **X**, and to

exclude any portion of the sub-box **X** that violates the *f_bar* inequality.

26. (Previously presented) The apparatus of claim 21, wherein if the sub-box **X** is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1,\ldots,n$):

the global optimizer is configured to,

determine a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1,\ldots,n$), and to

remove from consideration any sub-box for which $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that the sub-box does not include an extremum of $f(\mathbf{x})$; and

the term consistency mechanism is configured to,

apply term consistency to each component $g_i(\mathbf{x})=0$ ($i=1,\ldots,n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the sub-box **X**, and to

exclude any portion of the sub-box **X** that violates any component of $\mathbf{g}(\mathbf{x})=\mathbf{0}$.

27. (Previously presented) The apparatus of claim 21, wherein if the sub-box **X** is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1,\ldots,n$):

the global optimizer is configured to,

determine diagonal elements $H_{ii}(\mathbf{x})$ ($i=1,\ldots,n$) of the Hessian of the function $f(\mathbf{x})$, and to

remove from consideration any sub-box for which $H_{ii}(\mathbf{x})$ a diagonal element of the Hessian over the sub-box **X** is always negative, indicating that the function f is not convex over the sub-box **X** and consequently does not contain a global minimum within the sub-box **X**; and

the term consistency mechanism is configured to,

apply term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1,\ldots,n$) over the sub-box **X**, and to

exclude any portion of the sub-box **X** that violates a Hessian inequality.

28. (Previously presented) The apparatus of claim 21, wherein if the sub-box **X** is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1,\ldots,n$):

the global optimizer is configured to perform the Newton method, wherein performing the Newton method involves,

computing the Jacobian $\mathbf{J}(\mathbf{x},\mathbf{X})$ of the gradient of the function f evaluated with respect to a point $\mathbf{x}$ over the sub-box **X**,

computing an approximate inverse **B** of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$, and

using the approximate inverse **B** to analytically determine the system $\mathbf{Bg}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1,\ldots,n$); and

the term consistency mechanism is configured to,

apply term consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ ($i=1,\ldots,n$) for each variable x_i ($i=1,\ldots,n$) over the sub-box **X**, and to

exclude any portion of the sub-box **X** that violates a component.

29 (Canceled).

30. (Original) The apparatus of claim 21, wherein the global optimizer is configured to apply the Newton method to the John conditions.